

Information Sciences

Network system capacity: Towards integrating sensing, communication and control

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Received 21 June 2023; Revised 28 July 2023; Accepted 1 August 2023; Published online 6 November 2023

Network systems refer to a new generation of systems with integrated information perception, transmission and utilization capabilities through communication networks, which are adopted to achieve desirable objectives under physical and information related uncertainties and/or adversary conditions. In the information-rich era [1, 2], one of the fundamental issues is to exploit the limit of feedback control on dissipating such uncertainties in the scenarios of networked sensing and communication [3]. The existing information and control theories are difficult to reconcile due to different mathematical basis and conceptual paradigms from their aspects. It is challenging to effectively reveal the intrinsic triadic relation among networked sensing, communication, and feedback control. The concept of “network system capacity” is proposed to give a unified analytical expression for the integration of sensing, communication and control.

A typical architecture of network systems is shown in Figure 1, which consists of the controlled plant, sensor network, communication network, and feedback controllers. The state information of the controlled plant is collected through the sensor network, and transmitted to the estimator for achieving system state estimation. The estimation is passed to the controller to generate control commands, which are sent to the actuator through the communication network to realize the closed-loop control. The dynamics of the plant is $x[k+1] = a(x[k] + B[k]u^*[k]) + w[k]$, where $x[k]$, $u^*[k]$, and $w[k]$ denote the state, control input vector, and the noise of the plant, respectively, the scalar a is the system gain. The term $u^*[k] = \gamma[k]u[k]$, where $u[k] = K\hat{x}[k]$ is the designed control law with the feedback gain K and the estimate $\hat{x}[k]$ of $x[k]$, and the probability of successful transmission through the noisy channel $\gamma[k] \sim p_\gamma$, is of independent identical distribution (i.i.d.). The mean and variance of $\gamma[k]$ are denoted as μ_γ and σ_γ^2 . The sensing equation is $y[k] = C[k]G[k]x[k] + v[k]$, where $G[k]$ and $C[k]$ are the observation vector and indicator matrix of the activated sensors, respectively. The stochastic variables $w[k] \sim p_w$ and $v[k] \sim p_v$ are the white noises. The mean and variance of $v[k]$ are denoted as μ_v and σ_v^2 , respectively. Based on the aforementioned network system model, the “network system capacity” is derived from the following three aspects.

Firstly, communication networks offer competitive advantages in collaboration of the sensing and control processes, such as flexibility and scalability. To improve the collaborative sensing and control performance, it is necessary to further reveal the analytical relationship of channel capacity and sensing/control performance. The channel capacity is defined to be the maximum rate at which information can be transmitted through a channel [4], i.e., $C_c = W \log_2(1 + \mu_b^2/\sigma_b^2)$, where W is the bandwidth. It portrays the theoretical limit of the amount of sensing data $G[k]x[k]$ and control data $u[k]$ that can be transmitted over the noisy channels. The channel capacity thus constrains the sensing and control processes in the loop of the system.

Secondly, networked sensing enables the collaboration and data interaction of multiple sensors to achieve continuous and efficient monitoring of the system. It is a fundamental process for information acquisition and fusion through sensor networks, and plays an important role in state/parameter estimation and control of network systems. However, it is corrupted by the

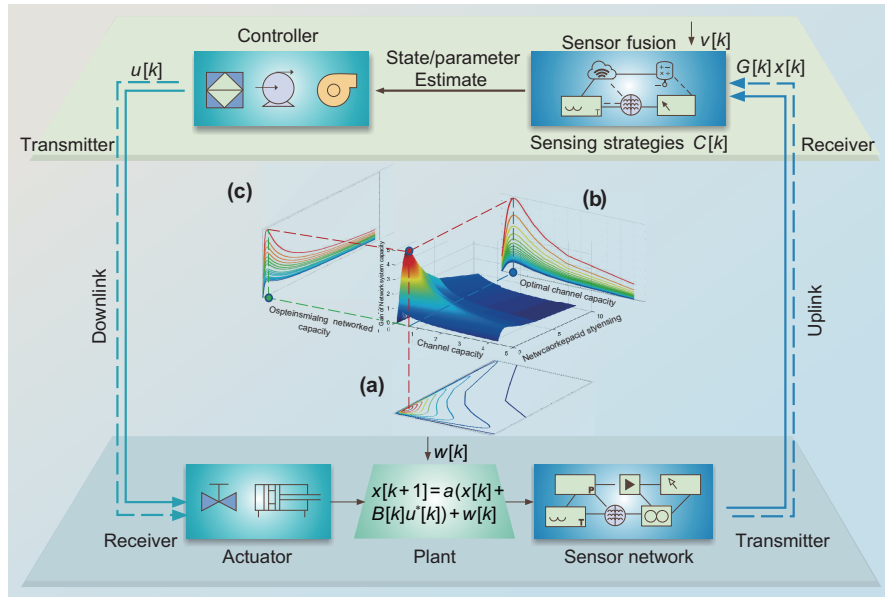


Figure 1 Network system architecture. (a) “Boomerang” like relationship between channel capacity and networked sensing capacity. (b) Relationship between the gain of network system capacity and channel capacity. (c) Relationship between the gain of network system capacity and networked sensing capacity.

inevitable uncertainties such as ambient noise, interference and multi-path fading [5,6]. Thus, the new concept called networked sensing capacity $C_s = 1/\sigma_v^2$ is defined to measure the estimation performance and sensor measurements under a given network topology.

Finally, feedback control is the process of returning the output information of the system to the input side, and using the deviation of the output and input information to steer the state of the controlled plant to a predetermined stable or equilibrium trajectory [7]. To describe the capability of controllers to stabilize a system, the concept of control capacity is introduced by [8, 9]. However, in the network system as in Figure 1, the networked sensing capacity and channel capacity limit the accuracy of the state estimation and successful transmission rate of the control command, respectively. Those uncertainties from sensing and communication processes induce challenges in capturing the fundamental limits of the control performance. Inspired by [8], the network system capacity is expressed based on the second-moment stability in one-step from the perspective of feedback control with the considered uncertainties, i.e., $C_N = \max_{u[k]} -\frac{1}{2} \log \mathbb{E} \left[\left(\frac{x[k]}{x[k-1]} \right)^2 \right]$ for all $k \in \mathbb{R}^+$, where the maximum can be obtained by the proper design of feedback gain K similarly as [8]. Thus, the network system capacity is expressed as

$$C_N = \frac{1}{2} \log \left[1 + \frac{C_s}{\alpha_0 \cdot 2^{C_c/W} + (2^{C_c/W} - 1) \cdot C_s} \right]$$

where α_0 is a constant determined by the initial state $x[0]$, and C_N characterizes the capability of the controller to stabilize the plant under sparse networked sensing and lossy communication, i.e., the uncertainty dissipation ability under the optimal system feedback gain.

Figure 1 reveals the gain of network system capacity composed of $\frac{\partial C_N}{\partial C_c}$ and $\frac{\partial C_N}{\partial C_s}$ under the influence of C_s and C_c . Inset (a) shows the “boomerang” like relationship of C_s and C_c . It reveals that C_s decreases with respect to C_c for a given gain of network system capacity. Insets (b) and (c) reflect that the gain of network system capacity varies with the incremental change of C_s and C_c . The top point corresponds to the optimal gain of network system capacity. The optimal values of channel capacity C_c and networked sensing capacity C_s can also be easily found, respectively. When C_c and C_s are smaller than their optimums, the gain of network system capacity increases very fast. However, when C_c and C_s are larger than their optimums, the gain of network system capacity drops greatly. It means the network system capacity is becoming saturated.

The network system capacity is tightly related to networked sensing capacity and channel capacity and the intrinsic triadic relationship is exploited from the perspective of control. When considering the general linear systems with multiple state

variables, an expression of network system capacity can be obtained in a similar way. When considering multiple controlled plants, there exist complex communication and sensing resource competitions among different loops. It then yields more difficulties in giving a unified analytical expression for the integration of sensing, communication and control. Of course, the relationship can also be explored from the perspective of networked sensing capacity and/or channel capacity. From the perspective of sensing, the control force makes the network system obtain richer measurements and more accurate estimation. The channel capacity then limits the ability of interacting among sensors, the controller and actuator. From the perspective of communication, efficient sensing of the communication environment is required to accurately identify interference to achieve optimized network capacity [10]. Besides, control theory can also be developed for congestion avoidance and quality of service (QoS) improvements. Based on different perspectives, the intrinsic coupling among sensing, communication and control can be explored comprehensively so that the overall system performance could be optimized by integrating appropriate sensing mechanisms, communication protocols and control laws.

Funding

This work was supported by the National Natural Science Foundation of China (92167205).

Author contributions

The author thanks Dr. Ling Lyu, Mr. Haifan Su, and Mr. Zhiduo Ji for the simulation and illustration, and thanks Mr. Jinglong Zhang and Mr. Lei Xu for the constructive discussion.

Conflict of interest

The author declares no conflict of interest.

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